Growth Chamber Experiments

Replications

Chamber 1

	A		
B	1	2	3
1	1	3	5
2	2	4	6

Chamber 2

	A		
B	1	2	3
1	1	3	5
2	2	4	6

Chamber 3

	A		
B	1	2	3
1	1	3	5
2	2	4	6

Model: $Y_{ijk} = \mu + C_i + \delta_{(i)} + A_j + B_k + AB_{jk} + \varepsilon_{ijk}$

$Y_{ijk} = \mu + C_i + \delta_{(i)} + A_j + CA_{ij} + B_k + CB_{ik} + AB_{jk} + CAB_{ijk}$

Growth Chamber Experiments

Replications

Source	df	3 R i	3 F j	2 F k	EMS	
C_i	2	1	3	2	$\sigma^2 + 6\sigma^2_\delta + 6\sigma^2_C$	
$\delta_{(i)}$	0	1	3	2	$\sigma^2 + 6\sigma^2_\delta$	
A_j	2	3	0	2	$\sigma^2 + 2\sigma^2_{CA} + 6\Phi[A]$	*
B_k	1	3	3	0	$\sigma^2 + 3\sigma^2_{CB} + 9\Phi[B]$	*
AB_{jk}	2	3	0	0	$\sigma^2 + 3\Phi[AB]$	
ε_{ijk}	10	1	0	0	$\sigma^2 + \sigma^2_{ABC}$	*

* Blue variance terms are pooled into error. They should be ignored (disappear) when defining appropriate F tests.

Growth Chamber Experiments
Treatments

Temperature 1

	A		
B	1	2	3
1	1	3	5
2	2	4	6

Temperature 2

	A		
B	1	2	3
1	1	3	5
2	2	4	6

Model: $Y_{ijk} = \mu + T_i + \delta_{(i)} + A_j + TA_{ij} + B_k + TB_{ik} + AB_{jk} + \varepsilon_{ijk}$

Growth Chamber Experiments
Treatments

Source	df	2 F i	3 F j	2 F k	EMS
T_i	1	0	3	2	$\sigma^2 + 6\sigma^2_\delta + 6\Phi[T]$
$\delta_{(i)}$	0	1	3	2	$\sigma^2 + 6\sigma^2_\delta$
A_j	2	2	0	2	$\sigma^2 + 4\Phi[A]$
TA_{ij}	2	0	0	2	$\sigma^2 + 2\Phi[TA]$
B_k	1	2	3	0	$\sigma^2 + 6\Phi[B]$
TB_{ik}	1	0	3	0	$\sigma^2 + 3\Phi[TB]$
AB_{jk}	2	2	0	0	$\sigma^2 + 2\Phi[AB]$
ε_{ijk}	2	0	0	0	$\sigma^2 + \Phi[TAB]$

Growth Chamber Experiments
Treatments

Temperature 1

B \ A	1	2	3
1	1	3	5
2	2	4	6

Temperature 2

B \ A	1	2	3
1	1	3	5
2	2	4	6

Temperature 1

B \ A	1	2	3
1	1	3	5
2	2	4	6

Temperature 2

B \ A	1	2	3
1	1	3	5
2	2	4	6

Model: $Y_{ijkl} = \mu + T_i + R_{(i)j} + \delta_{(ij)} + A_k + TA_{ik} + B_l + TB_{il} + AB_{kl} + \varepsilon_{ijkl}$

Growth Chamber Experiments
Treatments

Source	df	2 F i	2 R j	3 F k	2 F l	EMS
T_i	1	0	2	3	2	$\sigma^2 + 6\sigma^2_\delta + 6\sigma^2_R + 12\Phi[T]$
$R_{(i)j}$	2	1	1	3	2	$\sigma^2 + 6\sigma^2_\delta + 6\sigma^2_R$
$\delta_{(ij)}$	0	1	1	3	2	$\sigma^2 + 6\sigma^2_\delta$
A_k	2	2	2	0	2	$\sigma^2 + 8\Phi[A]$
TA_{ik}	2	0	2	0	2	$\sigma^2 + 4\Phi[TA]$
B_l	1	2	2	3	0	$\sigma^2 + 12\Phi[B]$
TB_{il}	1	0	2	3	0	$\sigma^2 + 6\Phi[TB]$
AB_{kl}	2	2	2	0	0	$\sigma^2 + 4\Phi[AB]$
$\varepsilon_{(i)jkl}$	11	1	1	0	0	σ^2

Unbalanced Designs

Missing Data

Treatment	**Blocks**				Total	Mean
	1	2	3	4		
1 $(NH_4)_2SO_4$	32.1	35.6	41.9	35.4	145.0	36.3
2 NH_4NO_3	30.1	31.5	37.1	30.8	129.5	32.4
3 $CO(NH_2)_2$	.	27.1	33.8	31.1	92.0	30.7
4 $Ca(NO_3)_2$	24.1	33.0	35.6	31.4	124.1	31.0
5 $NaNO_3$	26.1	31.0	33.8	31.9	122.8	30.7
6 Control	23.2	24.8	26.7	26.7	101.4	25.4
Total	135.6	183.0	208.9	187.3	714.8	
Mean	27.1	30.5	34.8	31.2		31.1

Unbalanced Designs

Estimating Missing Data

Treatment	Blocks				Total
	1	2	3	4	
1 $(NH_4)_2SO_4$	32.1	35.6	41.9	35.4	145.00
2 NH_4NO_3	30.1	31.5	37.1	30.8	129.50
3 $CO(NH_2)_2$	.	27.1	33.8	31.1	92.00
4 $Ca(NO_3)_2$	24.1	33.0	35.6	31.4	124.10
5 $NaNO_3$	26.1	31.0	33.8	31.9	122.80
6 Control	23.2	24.8	26.7	26.7	101.40
Total	135.6	183.0	208.9	187.3	714.8

$$\hat{Y}_{ij} = \frac{(IY_{i.} + JY_{.j} - Y_{..})}{(I-1)(J-1)} = \frac{6(92)+4(135.6)-714.8}{(4-1)(6-1)} = 25.3067$$

Unbalanced Designs

SS Adjustment

Source	DF	SS	Mean Square	F Value	Pr > F
Block	3	193.4976444	64.4992148	21.21	<.0001
Treatment	5	256.4335926	51.2867185	16.87	<.0001

$$SS_{Blk} = \frac{[Y_{i.} - (J-1)\hat{Y}_{ij}]^2}{J(J-1)} = \frac{[92.0-(4-1)25.3067]^2}{4(4-1)} = 21.5472$$

$$SS_{Trt} = \frac{[Y_{.j} - (I-1)\hat{Y}_{ij}]^2}{I(I-1)} = \frac{[135.6-(6-1)25.3067]^2}{6(6-1)} = 2.7402$$

Source	DF	SS	Mean Square	F Value	Pr > F
Block	3	171.9504444			
Treatment	5	253.6934445			

Unbalanced Designs

Adjusted ANOVA

Source	DF	SS	Mean Square	F Value	Pr > F
Block	3	171.9504444	54.4658814	16.72	<.0001
Treatment	5	253.6934445	44.8667185	13.77	<.0001
Error	14	45.6112222	3.2579444		
Total	22	495.5424592			

SAS Analysis

```
data c;
input blk trt yield @@;
cards;
1  1  32.1  2  1  35.6  3  1  41.9  4  1  35.4
1  2  30.1  2  2  31.5  3  2  37.1  4  2  30.8
1  3    .   2  3  27.1  3  3  33.8  4  3  31.1
1  4  24.1  2  4  33.0  3  4  35.6  4  4  31.4
1  5  26.1  2  5  31.0  3  5  33.8  4  5  31.9
1  6  23.2  2  6  24.8  3  6  26.7  4  6  26.7
;
proc glm data=c;
  class blk trt;
  model yield = blk trt;
  lsmeans trt / t;
run;
```

Unbalanced Designs

SAS Analysis

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	8	418.0079082	52.2509885	16.04	<.0001
Error	14	45.6112222	3.2579444		
Total	22	463.6191304			

Source	DF	**Type I SS**	Mean Square	F Value	Pr > F
Block	3	164.3144638	54.7714879	16.81	<.0001
Treatment	5	253.6934444	50.7386889	15.57	<.0001

Source	DF	**Type III SS**	Mean Square	F Value	Pr > F
Block	3	171.9504444	57.3168148	17.59	<.0001
Treatment	5	253.6934444	50.7386889	15.57	<.0001

Unbalanced Designs

Adjusted Standard Error of the Mean

$$S_{\bar{x}} = \sqrt{\frac{s^2}{r}} \times \sqrt{1 + \frac{t}{(t-1)(r-1)}}$$

Barley Example:

$$S_{\bar{x}} = \sqrt{\frac{3.2579}{4}} \times \sqrt{1 + \frac{6}{(6-1)(4-1)}} = .9024(1.1832) = 1.068$$

Unbalanced Designs

Least Square Means and Standard Errors

trt	yield LSMEAN	Standard Error	Pr > |t|
1	36.2500000	0.9024888	<.0001
2	32.3750000	0.9024888	<.0001
3	29.3266667	1.0678392	<.0001
4	31.0250000	0.9024888	<.0001
5	30.7000000	0.9024888	<.0001
6	25.3500000	0.9024888	<.0001

Treatment 3 arithmetic mean = 30.66

Mean Comparisons

Standard Error of a Mean Difference

$$s_{\bar{d}} = \sqrt{s_p^2\left(\frac{2}{r} + \frac{t}{r(r-1)(t-1)}\right)}$$

Barley Example:

$$s_{\bar{d}} = \sqrt{3.258\left(\frac{2}{4} + \frac{6}{4(4-1)(6-1)}\right)} = 1.398$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_{\bar{x}_1 - \bar{x}_2}} = \frac{\bar{d}}{s_{\bar{d}}} = \frac{36.25 - 29.32}{1.398} = 4.592$$

P > |t| =0.0002

Barley Example

GLM LSMEANS Comparisons

Least Squares Means for effect trt
Pr > |t| for H0: LSMean(i)=LSMean(j)

Dependent Variable: yield

i/j	1	2	3	4	5	6
1		0.0089	0.0002	0.0011	0.0007	<.0001
2	0.0089		0.0468	0.3081	0.2105	<.0001
3	0.0002	0.0468		0.2446	0.3426	0.0130
4	0.0011	0.3081	0.2446		0.8027	0.0006
5	0.0007	0.2105	0.3426	0.8027		0.0009
6	<.0001	<.0001	0.0130	0.0006	0.0009	

Barley Example

MIXED LSMEANS Comparisons

Differences of Least Squares Means

Effect	trt	_trt	Estimate	Standard Error	DF	t Value	Pr > |t|
trt	1	2	3.8750	1.2768	14	3.03	0.0089
trt	1	3	6.8421	1.3975	14	4.90	0.0002
trt	1	4	5.2250	1.2768	14	4.09	0.0011
trt	1	5	5.5500	1.2768	14	4.35	0.0007
trt	1	6	10.9000	1.2768	14	8.54	<.0001
trt	2	3	2.9671	1.3975	14	2.12	0.0520
trt	2	4	1.3500	1.2768	14	1.06	0.3083
trt	2	5	1.6750	1.2768	14	1.31	0.2107
trt	2	6	7.0250	1.2768	14	5.50	<.0001
trt	3	4	-1.6171	1.3975	14	-1.16	0.2666
trt	3	5	-1.2921	1.3975	14	-0.92	0.3709
trt	3	6	4.0579	1.3975	14	2.90	0.0116
trt	4	5	0.3250	1.2768	14	0.25	0.8028
trt	4	6	5.6750	1.2768	14	4.44	0.0006
trt	5	6	5.3500	1.2768	14	4.19	0.0009

Unbalanced Designs

Mean Comparisons

		B 1	B 2	Mean
A	1	7, 9	5	7
A	2	8	4, 6	6
Mean		8	5	

		B 1	B 2	Mean
A	1	8	5	6.5
A	2	8	5	6.5
Mean		8	5	

Unbalanced Designs

Mean Comparisons

		B: 1	B: 2
A	1	$7 = \mu + \alpha_1 + \beta_1$ $9 = \mu + \alpha_1 + \beta_1$	$5 = \mu + \alpha_1 + \beta_2$
A	2	$8 = \mu + \alpha_2 + \beta_1$	$4 = \mu + \alpha_2 + \beta_2$ $6 = \mu + \alpha_2 + \beta_2$

Unbalanced Designs

Mean Comparisons

Mean Difference $A_1 - A_2$

$$= 1/3\ [(\alpha_1 + \beta_1) + (\alpha_1 + \beta_1) + (\alpha_1 + \beta_2)] - 1/3[(\alpha_2 + \beta_1) + (\alpha_2 + \beta_2) + (\alpha_2 + \beta_2)]$$

$$= (\alpha_1 - \alpha_2) + 1/3\ (\beta_1 - \beta_2)$$

Intended vs. Actual Hypothesis

H_0: $\alpha_1 - \alpha_2 = 0$

H_0: $\alpha_1 - \alpha_2 + 1/3\ (\beta_1 - \beta_2) = 0$